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Admissible Inversion on Γ_1 Non-Deranged Permutations

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Some further theoretic properties of scheme called Γ_1 non deranged permutations, the permutation which fixes the first element in the permutations were identified and studied in relation to admissible inversion in this paper. This was done first through some computation on this scheme using prime number $p \geq 5$, the admissible inversion descent $aid(\omega_{p-1})$ is equi-distributed with descent number $des(\omega_{p-1})$ and also showed that the admissible inversion set $Ai(\omega_i)$ and admissible inversion set $Ai(\omega_{p-i})$ are disjoint.

Keywords: Inversion numbers; admissible inversion; descent number and $\Gamma_1 -$ non deranged permutations.

1 Introduction

An Inversion of $(i, j) \in Inv(f)$ is admissible if either $f(j) < f(j+1)$ or $f(j) > f(k)$ for some $i < k < j$ which is denoted as $Ai(f)$ and the number of admissible inversion f denoted by $ai(f) = |Ai(f)|$ .Permutation statistic has a long history and has grown at rapid pace in the last few decades

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the subject originated in early 19h century by the work of [1] until [2] extensive study which become an established discipline of Mathematics. In the last three decades much progress has been made in discovering and analyzing new statistics see for example [3,4,5,6,7,8] studied the representation of Γ_1 -non deranged permutation group $G_P^{\Gamma_1}$ via group character and also established that the character of every $\omega_i \in G_P^{\Gamma_1}$ is equal to one if $\omega_i = e$ otherwise p. Also the non standard Young tableaux of Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ has been studied by [9], they established that the Young tableaux of this permutation group is non standard. [10] studied pattern popularity in Γ_1-non deranged permutations they establish algebraically that pattern τ_1 is the most popular and pattern τ_3, τ_4 *and* τ_5 are equipopular in $G_P^{\Gamma_1}$ they further provided efficient algorithms and some results on popularity of patterns of length-3 in $G_P^{\Gamma_1}$. [11] studied the Fuzzy ideal of function *f* Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ and established that it is one side fuzzy (only right fuzzy but not left) also the α-level cut of *f* coincides with $G_P^{\Gamma_1}$ if $\alpha = \frac{1}{p}$. [12] studied ascent on Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ in which recursion formula for generating Ascent number, Ascent bottom and Ascent top was develop and also observe that $Asc(\omega_i)$ union $Asc(\omega_{p-1})$ is equal to $Asc(\omega_1)$. [13] provide very useful theoretical properties of Γ_1-non deranged permutation s in relation to excedance and shown that the excedance set of all ω_i in $G_P^{\Gamma_1}$ such that $\omega_i \neq e$ is $\frac{1}{2}(p-1)$. [14] established that the intersection of descent set of all Γ_1-non derangement is empty, also observed that the descent number is strictly lessone [15] established that inversion number and major index are not equidistributed in Γ_1-non deranged permutations and also established that the difference between sum of the major index and sum of the inversion number is equal to sum of descent number in Γ_1-non deranged permutations. [16] studied standard representation of Γ_1-non deranged permutations and also identified relation to ascent block by partitioning the permutation set in which a recursion formula for generating maximum number of block and minimum number of block were develop and it is also observed $ar(\omega_i)$ that is equidistributed with $asc(\omega_i)$ for any arbitrary permutation group. [17] established that in Γ_1-non deranged permutations, the radius of a graph of any ω_1 is zero, the graph of any $\omega_i \in G_p^{\Gamma_1}$ is null, and by restricting 1, the graph of ω_{p-1} is complete. [18] established that the Right embracing number of Γ_1 -non deranged permutations of ω_i $\mathrm{Re}\,s(\omega_i)$ is equidistributed with the Left embracing $Les(\omega_i)$ and then $\mathrm{Re}\,s(\omega_i)$ is equidistributed with $\mathrm{Re}\,s(\omega_{p-i})$ and also observed that the height of weighted motzkin path of ω_i is the same as the height of weighted motzkin path of $\omega_{p-des(\omega_i)}$ [19] Investigated some algebraic theoretic properties of fuzzy set on $G_p^{'}$ using constructed membership function of fuzzy set on $G_p^{'}$ and established the result for algebraic operators of fuzzy set on $G_p^{'}$ which are algebraic sum, algebraic product, bounded sum and bounded difference and also constructed a relationship between the operators and fuzzy set on $G_p^{'}$. More recently [20] studied partition block coordinate statistics on Γ_1-non deranged permutations and observed that left opener bigger block $lobTC(\omega_i)$ is equidistributed with right opener bigger block $robTC(\omega_i)$.Hence we will in this paper show that admissible inversion set $Ai(\omega_i)$ and admissible inversion set $Ai(\omega_{p-i})$ are disjoint we also show that $aid(\omega_{p-1})$ (admissible inversion descent) is equal to $des(\omega_{p-1})$ (descent number).

2 Preliminaries

Definition 2.1 [15]

Let Γ be a non empty set of prime cardinality $p \geq 5$ such that $\Gamma \subset N$ A bijection ω on Γ of the form

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p \\ 1 & (1+i)_{mop} & (1+2i)_{mop} & . & . & . & (1+(p-1)i)_{mop} \end{pmatrix}$$

is called a Γ_1 -non deranged permutation. We denoted G_p to be the set of all Γ_1 -non deranged permutations. $G_7 = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$ is the set of all Γ_1 -non deranged permutations where $p = 7$

By definition 2.1, G_7 is generated as follows

$$\omega_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$$

$$\omega_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 5 & 7 & 2 & 4 & 6 \end{pmatrix}$$

$$\omega_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 7 & 3 & 6 & 2 & 5 \end{pmatrix}$$

$$\omega_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 5 & 2 & 6 & 3 & 7 & 4 \end{pmatrix}$$

$$\omega_5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 6 & 4 & 2 & 7 & 5 & 3 \end{pmatrix}$$

$$\omega_6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 7 & 6 & 5 & 4 & 3 & 2 \end{pmatrix}$$

Definition 2.2 [15]

The pair G_p and the natural permutation composition forms a group which is denoted as $G_P{}^{\Gamma_1}$ .This is a special permutation group which fixes the first element of Γ.

Definition 2.3 [8]

An inversion of permutation $f = \begin{pmatrix} 1 & 2 & 3 & . & . & . & n \\ f(1) & f(2) & f(3) & . & . & . & f(n) \end{pmatrix}$ is a pair (i, j) such that $i < j$ and $f(i) > f(j)$ .The inversion set of f , denoted as $Inv(f)$, is given by

$Inv(f) = \{(i,j) : 1 \le i < j \le n \text{ and } f(i) > f(j)\}$, the inversion number of f , denoted by $inv(f) = |Inv(f)|$.

Example 2.1

For ω_4 in $G_5^{\Gamma_1}$

$$\omega_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 4 & 3 & 2 \end{pmatrix}$$

$$Inv(\omega_4) = \{(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)\}$$
$$inv(\omega_4) = 6$$

Definition 2.4 [21]

An Inversion of $(i, j) \in Inv(f)$ is admissible if either $f(j) < f(j+1)$ or $f(j) > f(k)$ for some $i < k < j$ which is denoted as $Ai(f)$ and the number of admissible inversion f denoted by $ai(f) = |Ai(f)|$
.

Example 2.2

For ω_3 in $G_5^{\Gamma_1}$

$$\omega_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 5 & 3 \end{pmatrix}$$
$$Inv(\omega_3) = \{(2,3),(2,5),(4,5)\}$$
$$Ai(\omega_3) = \{(2,3),(2,5)\}$$
$$ai(\omega_3) = 2$$

Definition 2.5 [13]

A descent of permutation $f = \begin{pmatrix} 1 & 2 & 3 & . & . & . & n \\ f(1) & f(2) & f(3) & . & . & . & f(n) \end{pmatrix}$ is any positive $i < n$ (where i and n are positive integers and the current value is greater than the next), that is i is an descent of a permutation f if $f(i) > f(i+1)$.The descent set of f , denoted as

$Des(f)$, is given by $Des(f) = \{i : f(i) > f(i+1)\}$.The descent number of f , denoted as $des(f)$, is defined as the number of descent and is given by $des(f) = |Des(f)|$

Example 2.3

For ω_5 in $G_5^{\Gamma_1}$

$$\omega_5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 6 & 4 & 2 & 7 & 5 & 3 \end{pmatrix}$$

$$Des(\omega_4) = \{2,3,5,6\}$$
$$des(\omega_4) = 4$$

3 Main Results

In this section, we present some admissible inversion results of subgroup $G_P{}^{\Gamma_1}$ of S_p (Symmetry group of prime order with $p \geq 5$).

Proposition 3.1.

Let $\omega_1, \omega_{p-1} \in G_P{}^{\Gamma_1}$. Then the

$$ai(\omega_1) = ai(\omega_{p-1}) = 0$$

Proof.
Since the admissible inversion is a subset of inversion, and it is trivial that $inv(\omega_1) = 0$, hence $ai(\omega_1) = 0$. For $\omega_{p-1} = a_1, a_2, ...a_p$ it can be written as $\omega_{p-1} = 1, p, (p-1), ..., 2$ and from this we can see there is no $a_k > a_{k+1}$ or $a_k > a_m$ for some $j < m < k$ in the set $\{(i, j) : j > k \ and \ a_j > a_k\}$. Hence, $ai(\omega_{p-1}) = 0$

Remark 3.2

It is trivial that $Inv(\omega_1) = \phi$ since there is no $a_j > a_k$ for $j < k$ also $inv(\omega_1) = 0$ and admissible inversion is a subset of inversion therefore $ai(\omega_1) = 0$.

Corollary 3.3

Let $\omega_1, \omega_{P-1} \in G_P{}^{\Gamma_1}$. Then the

$$Ai(\omega_1) = Ai(\omega_{p-1}) = \phi$$

Proof.

By Proposition 3.1

$$ai(\omega_1) = ai(\omega_{p-1}) = 0$$

Since

$$ai(\omega_1) = |Ai(\omega_1)| = 0$$

And

$$ai(\omega_{p-1}) = \left|Ai(\omega_{p-1})\right| = 0$$

Therefore

$$Ai(\omega_1) = Ai(\omega_{p-1}) = \varnothing$$

Proposition 3.4

Let $G_P^{\Gamma_1}$ be a Γ_1-non derangement permutations, Then

$$Ai(\omega_2) = Inv(\omega_2)$$

Proof.

Given $\omega_2 = a_1, a_2, ...a_p$ The inversion of ω_i is the set of the pairs (j,k) with $j < k$ while the admissible inversion Ai is a subset of inversion in which $(j,k) \in Ai$ is $a_k < a_{k+1}$ or $a_k < a(m)$

for some $j < m < k.$ so $\left\{\frac{p+3}{2}, ...p\right\}$ are the members of a_k and are less than their respective

a_{k+1} except a_p therefore they are all in A_i except a_p but a_p takes the inversion $\left(\frac{p+1}{2}, p\right)$. Hence it is also admissible inversion of ω_2 and by embedding the A_k 's and A_p the result follows

Remark 3.5

From Proposition 3.4 we can deduce that $ai(\omega_2) = inv(\omega_2)$ and also that $ai(\omega_2)$ is equi-distributed with $inv(\omega_2)$

The admissible inversion descent of permutation f is the sum of the cardinality of admissible inversion and the cardinality of descent that is $aid(f) = ai(f) + des(f)$

Example

For ω_3 in $G_5^{\Gamma_1}$

$$\omega_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 5 & 3 \end{pmatrix}$$

$$Inv(\omega_3) = \{(2,3), (2,5), (4,5)\}$$

$$Ai(\omega_3) = \{(2,3), (2,5)\}$$

$$ai(\omega_3) = 2$$

$$Des(\omega_3) = \{2,4\}$$

$$des(\omega_3) = 2$$

$$aid(\omega_3) = 2 + 2 = 4$$

Proposition 3.6

Let $\omega_{p-1} \in G_P{}^{\Gamma_1}$. Then the

$$aid(\omega_{p-1}) = des(\omega_{p-1})$$

Proof.

From proposition 3.1 We have that the $ai(\omega_{p-1}) = 0,$ and recall the $aid(\omega_i) = ai(\omega_i) + des(\omega_i),$ hence the result follows.

Remark 3.7

The above Proposition implies that $aid(\omega_{p-1})$ is equi-distributed with $des(\omega_{p-1}),$ and also for ω_1 its descent number i.e. $des(\omega_1) = 0$ and $ai(\omega_1) = 0$. This implies that $aid(\omega_1) = des(\omega_1) = 0.$

Proposition 3.8

Let $G_P{}^{\Gamma_1}$ be a Γ_1-non derangement permutations, Then

$$Ai(\omega_i) \cap Ai(\omega_{p-i}) = \varnothing$$

Proof.

Given $\omega_i = a_1 a_2 ... a_{p-1} a_p$ the $\omega_{p-i} = a_1 a_p a_{p-1} ... a_2$. The inversion of ω_i is the set of the pairs (j,k) with $j < k$ such that $a_j > a_k$, but looking at ω_{p-1} and restricting a_1 it is the reverse of ω_i, therefore their inversion are disjoint. But admissible inversion is a subset of inversion. Hence,

$$Ai(\omega_i) \cap Ai(\omega_{p-i}) = \varnothing$$

Corollary 3.9

Suppose that $G_P{}^{\Gamma_1}$ is Γ_1-non derangement permutations, Then

$$Ai(\omega_{p-1}) = \bigcap_{i=1}^{p-1} Ai(\omega_i) = \varnothing$$

Proof.

From proposition 3.8 $Ai(\omega_i) \cap Ai(\omega_{p-1}) = \varnothing$ So, in this case we want to show for any $G_P{}^{\Gamma_1}$ there exist ω_i and ω_{p-i} since their intersection is $\varnothing$, then intersection of empty set with any set is also empty set, we already know that $G_P{}^{\Gamma_1}$ is defined for p is prime and $p \geq 5$, and we donate each set $G_P{}^{\Gamma_1} = \{\omega_1, ..., \omega_{p-1}\}$, from this we can see that for any $G_P{}^{\Gamma_1}$ we have atleast ω_1 and ω_{p-1}.

Proposition 3.10

Let $\omega_i \in G_5^{\Gamma_1}$. Then the

$$inv(\omega_{p-1}) = \sum_{i=1}^{p-1} ai(\omega_i) + 1$$

Proof.

Given $\omega_{p-1} \in G_5^{\Gamma_1}$, the $inv(\omega_{p-1}) = 2p-4.$, we already know that $G_5^{\Gamma_1} = \{\omega_1, ..., \omega_4\}$, where $ai(\omega_1) = ai(\omega_{p-1}) = 0$ and $ai(\omega_2) = p-2$ while $ai(\omega_3) = p-3$, summing them we have

$$\sum_{i=1}^{p-1} ai(\omega_i) = 0 + (p-2) + (p-3) + 0 = 2p-5$$

$$\sum_{i=1}^{p-1} ai(\omega_i) + 1 = 2p-5+1 = 2p-4$$

$$= inv(\omega_{p-1})$$

4 Conclusion

This paper has provided very useful theoretical properties of this scheme called Γ_1-non deranged permutations in relation to the admissible inversion. We have shown that admissible inversion set $Ai(\omega_i)$ and admissible inversion set $Ai(\omega_{p-i})$ are disjoint we also shown that $aid(\omega_{p-1})$ (admissible inversion descent) is equal to $des(\omega_{p-1})$ (descent number).

Further Research

G_p defined above is subgroup of extra ordinary group of group theory. One can find number of subgroups of order 4 using [22].

Competing Interests

Authors have declared that no competing interests exist.

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